EFFECT OF ANGULAR VIBRATION ON TRANSFER ALIGNMENT

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Effect of Angular Vibration on Transfer Alignment II

One of the findings of a recent study of transfer alignment methods conducted for the JSOW program office was that the alignment could be severely affected by angular vibration noise. At the time of this study, the root cause of this effect was not determined. When these results were presented at a JDAM Technical Interchange Meeting at China Lake last year, the validity of the data was questioned. A white paper was written sumarizing what was known about the problem at that time. That paper (Effect of Angular Vibration on Transfer Alignment I) is included here as an appendix. Since then, the JSOW program office has funded some further analysis in order to clarify the issue. As a result, it is now believed that the effect is well understood. In this paper we will explain the cause of the problem and present data supporting our conclusions.

In a velocity matching transfer alignment process, the slave (or missile) Inertial Navigation System (INS) compares its velocity solution to that of a master (or aircraft) INS. These two INS's are connected by a semi-rigid displacement. Hence the master velocity reference must be corrected by the vector product of the moment arm and the angular velocity of the aircraft. The measurement is the difference of the two. In simplified form

$$\mathbf{z} = \mathbf{v_S} - (\mathbf{v_M} + \mathbf{w} \times \mathbf{r}) \qquad (1)$$

where $\mathbf{v_S}$ = slave velocity,
$\mathbf{v_M}$ = master velocity,
$\mathbf{w}$ = angular velocity of aircraft with respect to the ECEF frame,
and $\mathbf{r}$ = displacement from master INS to slave INS (moment arm).

In a velocity integral transfer alignment, the measurement is the difference of the integrals of the two terms in equation 1.

In the current transfer alignment implementation, the aircraft computes only the $\mathbf{v_M}$ term. The missile must compute the moment arm correction. To do this it uses the value of $\mathbf{w}$ derived from it's gyro outputs. This value of $\mathbf{w}$ therefore includes the angular vibration noise. The angular vibration is thus corrupting the measurement through the $\mathbf{w}\mathbf{x}\mathbf{r}$ term. As described in the appendix, the effect can be dramatic.

A series of Monte Carlo runs was designed to test the above assumption. Each set of runs consists of 100 runs with random initialization errors, sensor errors and noise sequences. For the first case we ran with moment arms of

$$x = 4 \text{ ft}$$
$$y = 14 \text{ ft}$$
and $$z = -3 \text{ ft}.$$

There was no linear or angular noise modeled. For the second case the moment arms were set to zero and full angular noise was modeled (see appendix). For the last case the moment arms were again set to zero and both linear and angular noise modeled. These values may be compared to the results from the appendix for angular noise only with the above moment arms. The resulting CEP's at 260 sec after the end of the alignment are given in Table 1.

Table 1
CEP in ft after 260 sec drift.

Alignment Conditions	Velocity Match	Velocity Integral
Moment Arms No Noise	440	400
No Moment Arms Full Angular Noise	500	420
No Moment Arms Full Angular Noise Linear Noise	640	390
Moment Arms Full Angular Noise	2900	750

Note that that the values given in Table 2 of the appendix for the no noise case are actually for full angular noise on the x axis. The values in the table should agree with case 1 above. In addition, the runs which produced divergent solutions were rerun with no moment arms, resulting in nominal solutions.

These results clearly indicate the source of the problem and the magnitude of the errors it can cause. There are several things which could be done to reduce the alignment errors associated with angular vibration.

From a purely analytical viewpoint, the simplest and most effective solution would be to require the launch aircraft to compute the moment arm correction. This would eliminate most of the problem as the aircraft does not sense the missile vibration. However, this may not practical at this late date.

Another possibility would be to tune the alignment filter (measurement noise and process noise) to account for this measurement error. This will keep the nav solutions from becoming divergent but will impact the accuracy of a nominal alignment. It might also be possible to reduce this effect by filtering the slave gyro outputs before computing the moment arm corrections. This would also have an inpact on the accuracy of a nominal alignment.

In conclusion, any angular vibration of the missile will degrade the alignment quality. The magnitude of the degradation will depend on the magnitude of the angular vibration. The superiority of the velocity integral matching transfer alignment over the velocity matching transfer alignment is also clearly demonstrated.

If there are questions or comments, I may be reached at 619-939-3768 or DSN 437-3768.

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Effect Of Angular Vibration On Transfer Alignment

The effect on angular vibration on a transfer alignment has recently been investigated by NAWC-WD. The analysis consisted of a large number of Monte Carlo runs on the Code C2872 transfer alignment simulation. Both a master and slave INS are independently modeled in the simulation. A Kalman filter is incorporated in the slave INS model. Measurements are constructed from the differences between the two INS models. Moment arm and data senescence effects are modeled. Noise terms can be added to the measurements and to the gyro and accelerometer models. These consist of quantization noise, white noise, random walk, and first and second order Markov processes.

For the purposes of this analysis, second order Markov noise was added to the gyro outputs. The equation for the processes is

$$\ddot{X}(t) + 2\alpha\beta\,\dot{X}(t) + \beta^2 X(t) = \omega(t) \tag{1}$$

where $\omega(t)$ is a white noise process with autocorrelation

$$E[\omega(t)\,\omega(t+\tau)] = N\,\delta(\tau)\,. \tag{2}$$

N is the power spectral density in units of radians squared per seconds cubed.

The computer solution is based on the exact solutions

$$X(t+\tau) = \frac{e^{-\alpha\beta\tau}}{K}(K\cosh\beta K\tau + \alpha\sinh\beta K\tau)\,X(t) + \frac{e^{-\alpha\beta\tau}}{\beta K}\sinh\beta K\tau\,\dot{X}(t) + \frac{1}{\beta K}\int_0^\tau d\lambda\,\omega(t+\tau-\lambda)\,e^{-\alpha\beta\lambda}\sinh\beta K\lambda \tag{3}$$

$$\dot{X}(t+\tau) = \frac{e^{-\alpha\beta\tau}}{K}(K\cosh\beta K\tau - \alpha\sinh\beta K\tau)\,\dot{X}(t) - \frac{\beta}{K}e^{-\alpha\beta\tau}\sinh\beta K\tau\,X(t) + \frac{1}{K}\int_0^\tau d\lambda\,\omega(t+\tau-\lambda)\,e^{-\alpha\beta\lambda}(K\cosh\beta K\lambda - \alpha\sinh\beta K\lambda)$$

where $K = \sqrt{\alpha^2 - 1}\,.$

If we define the constants

$$A_1 = e^{-\alpha\beta\Delta t}\cosh\beta K\Delta t \tag{4}$$

$$A_2 = \frac{1}{K} e^{-\alpha\beta\Delta t}\sinh\beta K\Delta t$$

and the random variables

$$u_p = \frac{1}{\beta K}\int_0^{\Delta t} d\lambda\, \omega(t_p-\lambda)\, e^{-\alpha\beta\lambda}\sinh\beta K\lambda \tag{5}$$

$$v_p = \frac{1}{K}\int_0^{\Delta t} d\lambda\, \omega(t_p-\lambda)\, e^{-\alpha\beta\lambda}(K\cosh\beta K\lambda - \alpha\sinh\beta K\lambda)$$

we can write equations (3) as

$$x_{p+1} = (A_1 + \alpha A_2)x_p + \frac{1}{\beta}A_2\dot{x}_p + u_{p+1} \tag{6}$$

$$\dot{x}_{p+1} = -\beta A_2 x_p + (A_1 - \alpha A_2)\dot{x}_p + v_{p+1} \quad .$$

The second order statistics of u_p and v_p are

$$E[u_p^2] = \frac{N}{4\alpha\beta^3K^2}\left[K^2 - e^{-2\alpha\beta\Delta t}(\alpha^2\cosh 2\beta K\Delta t + \alpha K\sinh 2\beta K\Delta t - 1)\right] \tag{7}$$

$$E[v_p^2] = \frac{N}{4\alpha\beta K^2}\left[K^2 - e^{-2\alpha\beta\Delta t}(\alpha^2\cosh 2\beta K\Delta t - \alpha K\sinh 2\beta K\Delta t - 1)\right]$$

and

$$E[u_p v_p] = \frac{N}{2\beta^2K^2}e^{-2\alpha\beta\Delta t}\sinh^2\beta K\Delta t \quad ,$$

Since these equations are independent of time, we can replace equations (5) with a pair of discrete white sequences with the same statistics as above. This will give the same statistics for the discrete equations (6) as we would obtain by sampling the continuous process. We can obtain a pair of discrete sequences with the proper correlation by using two zero mean, unit variance sequences r_p and s_p , and setting

$$u_p = \sigma_u r_p \tag{8}$$

$$v_p = \frac{\sigma_{uv}^2}{\sigma_u} r_p + \sigma_v \left(1 - \frac{\sigma_{uv}^2}{\sigma_u^2 \sigma_v^2}\right)^{1/2} s_p$$

where

$$\sigma_u^2 = E[u_p^2] \quad , \quad \sigma_v^2 = E[v_p^2] \quad \text{and} \quad \sigma_{uv}^2 = E[u_p v_p] \ .$$

The above equations assume $\alpha \geq 1$. For $\alpha \leq 1$ we replace k with ik.

These equations were used to generate three second order Markov processes with the following characteristics:

Table 1. Angular Displacement

Axis	N	α	β (1/sec)	σ_θ (rad/sec)	σ_θ (deg)	f_0 (Hz)
x	1.34	0.1	27.5	0.35	0.5	4.4
y	37	0.475	40	0.7	1.0	5.6
z	62	0.8	40	0.7	1.0	3.8

$$f_0 = \frac{\beta}{2\pi}\sqrt{1-\alpha^2}$$

These three random angular displacements can be used as components of a random vector which represents a rotation of the slave body frame with respect to the master INS axes. Even though the random vector formed in this fashion will have zero mean, the transformation it generates will exhibit large excursions from the mean. To see this consider the second order expansion of a single axis rotation,

$$(R) = (R_0)\left[I + (\delta\theta) + \tfrac{1}{2}(\delta\theta)^2\right] . \tag{9}$$

where $(\delta\theta)$ is the matrix

$$(\delta\theta) = \begin{pmatrix} 0 & \delta\theta_z & -\delta\theta_y \\ -\delta\theta_z & 0 & \delta\theta_x \\ \delta\theta_y & -\delta\theta_x & 0 \end{pmatrix} \tag{10}$$

If we itterate this equation with a series of small angle rotations we find to second order

$$(R_N) = (R_0)\left[I + (\theta_N) + \frac{1}{2}(\theta_N)^2 + \sum_{K=1}^{N} (\delta\theta_K \times \theta_K)\right] \tag{11}$$

where

$$\underline{\theta}_N = \sum_{j=1}^{N} \underline{\delta\theta}_j$$

is a zero mean vector. The cross term in the above equation is the source of bias drift. To correct for this we add to each noise vector computation

$$\underline{\epsilon}_N = \frac{1}{2}(\underline{\theta}_N \times \underline{\delta\theta}_N)$$

before applying it to the gyro outputs. This produces zero mean noise on the output of the navigation attitude matrix.

A large number of Monte Carlo runs were performed using both velocity matching and velocity integral matching transfer alignments. The alignments were six minutes in duration followed by five minutes of drift. Sensor errors, initialization errors, and the noise sequences were randomized from run to run. Only angular noise was modeled. The following results were obtained:

Table 2. CEP at End of Drift (ft)

	full noise	1/2 amp noise	no noise
velocity match	2900	1420	1000 ← full noise on y axis
integrated velocity match	750	560	500

Full noise refers to the values in Table 1.

During a set of 100 Monte Carlo runs for velocity matching at full angular noise, the navigation solutions on two of the runs were observed to become extremely large. It was discovered that the nav solutions for these two runs had become divergent, even though the filter's covariance estimates were nominal. Figures 1 thru 5 show the noise input on the x axis, the roll error, the filter covariance, the x gyro bias, and the x velocity error for the first 150 seconds of one of the alignments. A turn occurs at 120 seconds. The horizontal error at the end of the nav portion of the run was 60,000 ft.

I am unable to give a logical explanation of the problem at this time. However, it seems to be closely associated with the alignment process. If the filter is turned off, the problem disappears and nav is normal. The problem also disappears when the identical run is done with a velocity integral transfer alignment. A velocity matching transfer alignment also produces a nominal alignment if process noise is added to the tilt states or the measurement noise is increased. The possibility of an error in the C2872 simulation is still a possibility which will be investigated further as time permits.

Clearly the angular noise model used in the analysis was extremely conservative. I would not expect the real environment to be this severe. However, the whole point of the analysis was that the environment under which the transfer alignment must be performed is largely unknown, and a poorly modeled filter can cause large problems. The analysis also demonstrates the robustness of the velocity integral alignment as compared to the velocity matching alignment.

If there are questions, I may be contacted at 619-939-3768.

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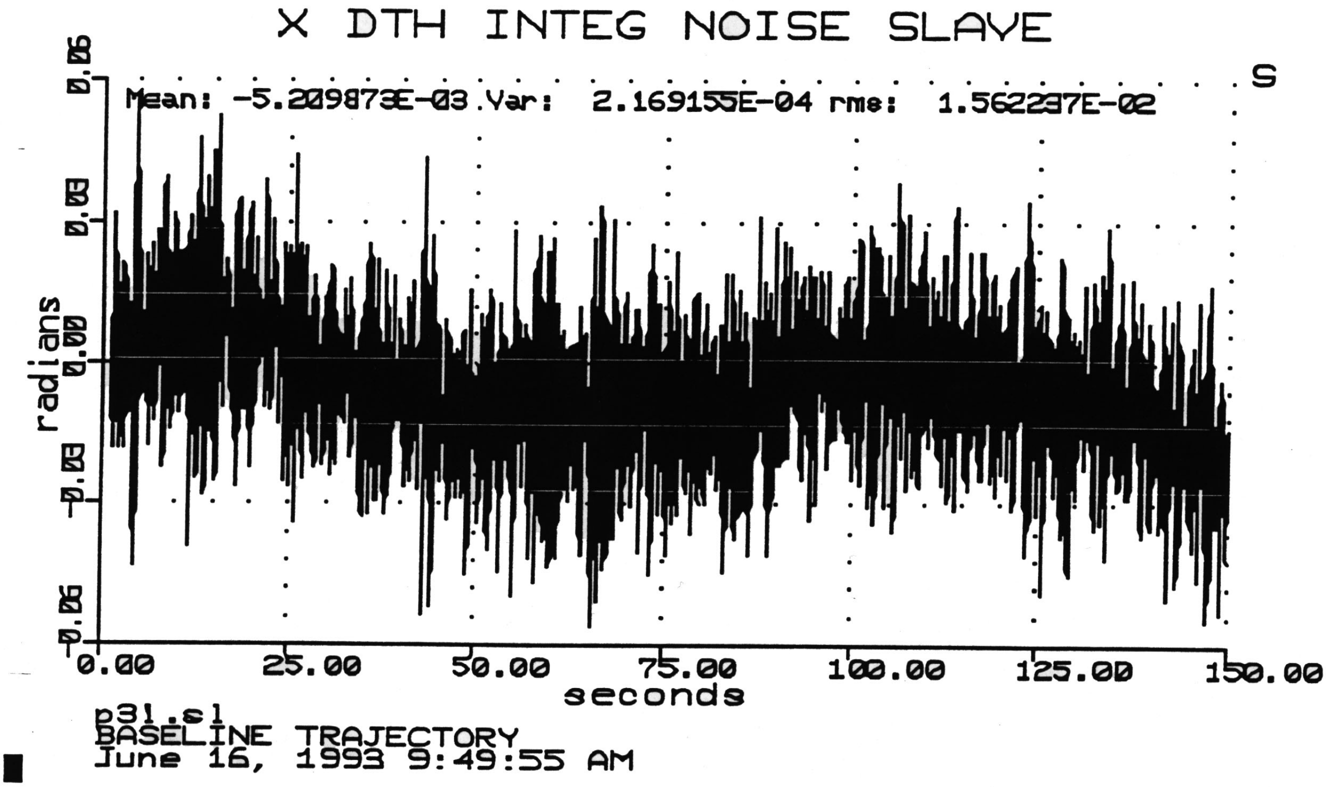

Figure 1

X axis Noise

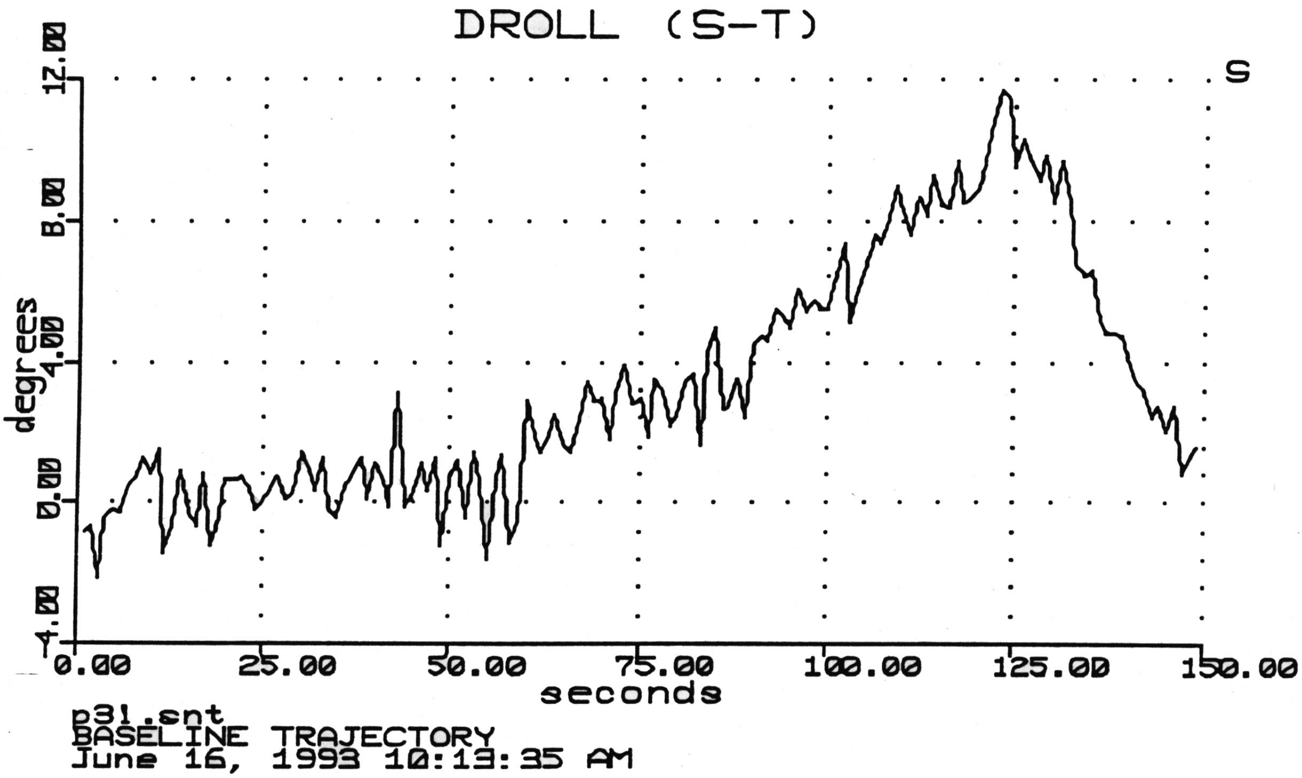

Figure 2

Navigation Roll Error

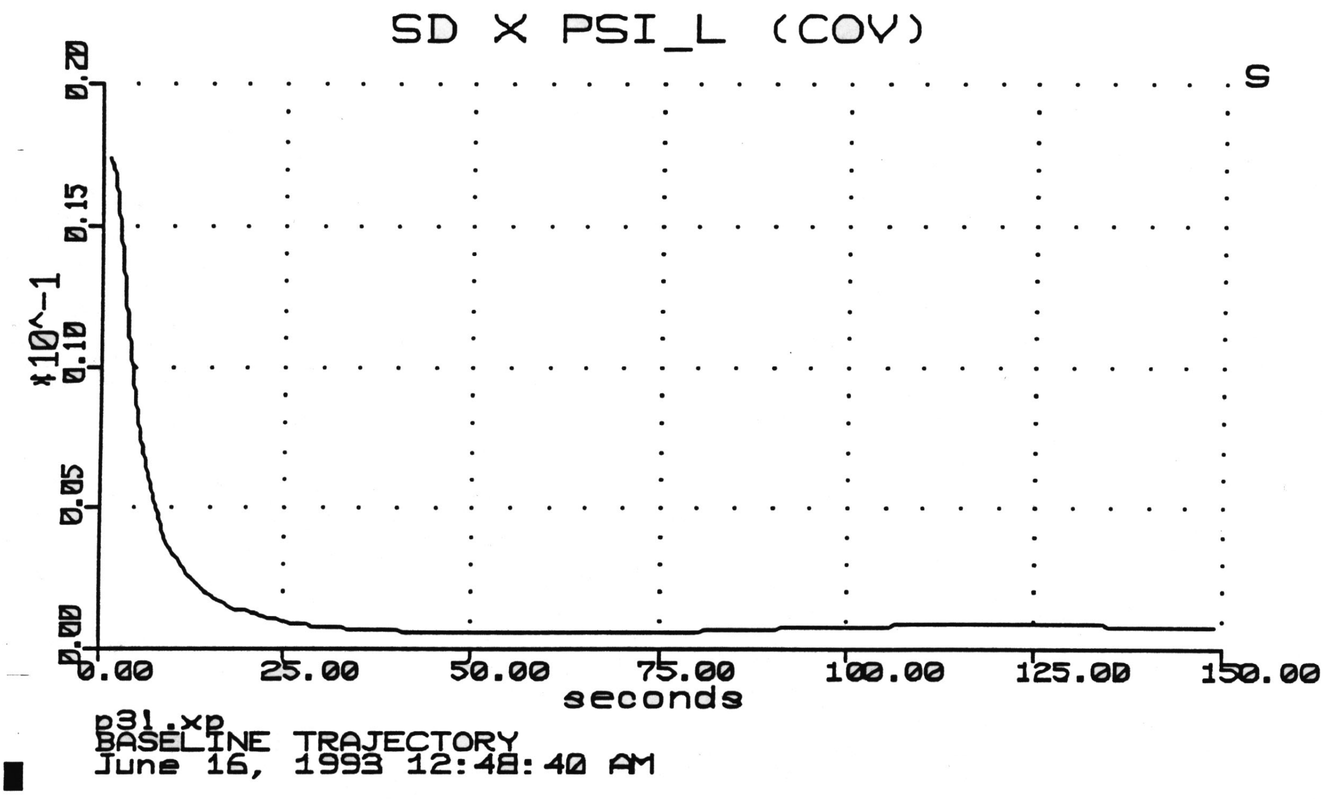

Figure 3

Covariance Estimate

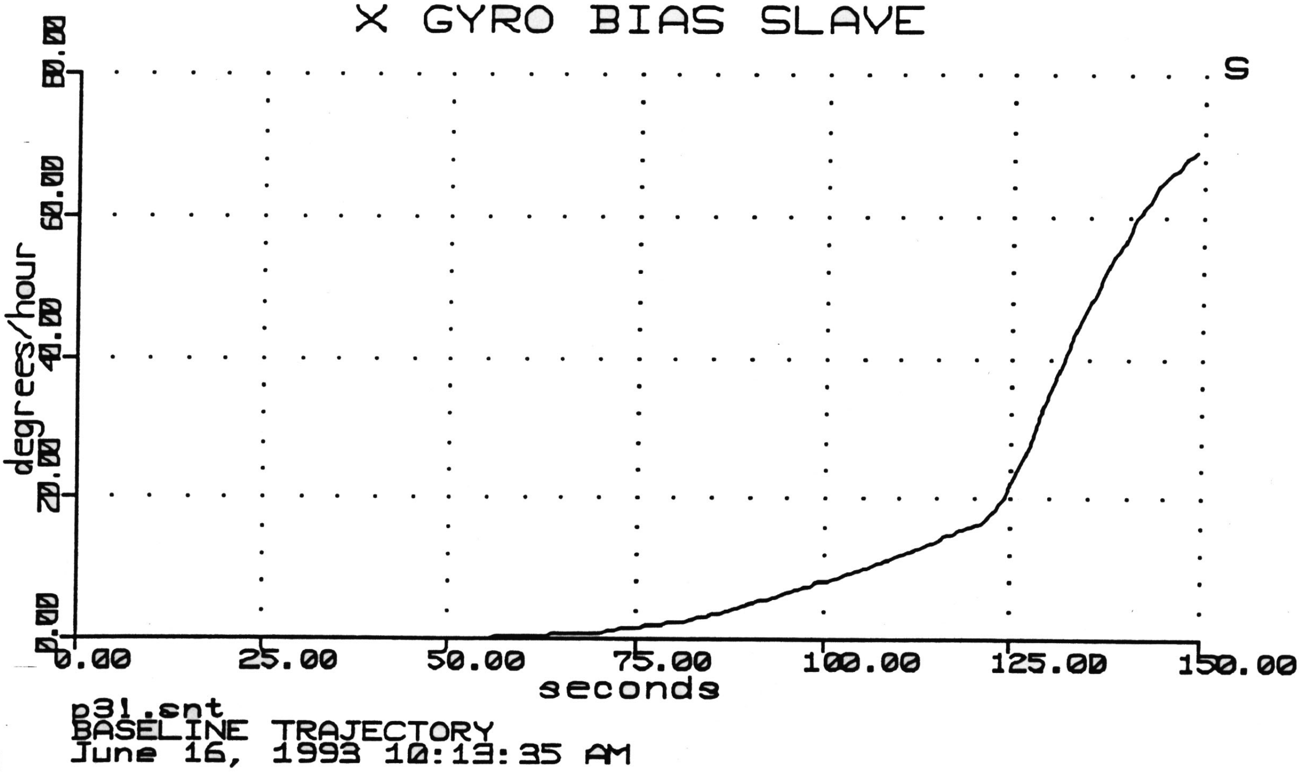

Figure 4

X Gyro Bias

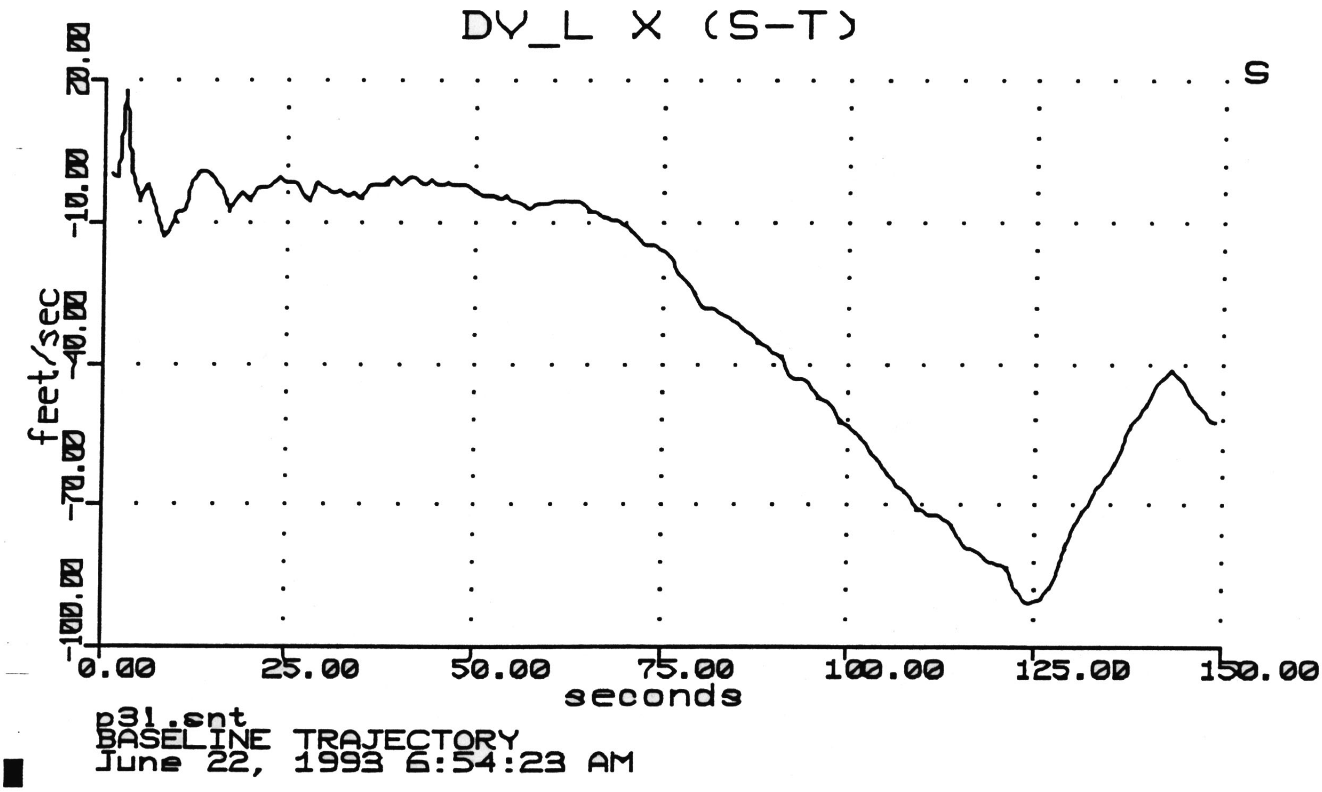

Figure 5

X Velocity Error

Note - Coordinate Frames

The original JSOW ICD specified a wander azimuth frame based on the ENU coordinate frame for the transfer alignment interface. This was done in order to be compatible with the Navy's standard navigator. During the last six months some informal changes have crept into the draft ICD. The coordinate frame has changed from ENU to NED. A meeting was held recently at China Lake to discuss the issue. In view of changed circumstances it was decided to formalize this change. The ICD interface will be a wander azimuth frame based on an NED frame. This will be compatible with the proposed changes to Mil Std 1760.